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$$f(n) = \frac{1}{n \ln n},$$

$$f'(n) = -\frac{1}{(n \ln n)^2}\left(\ln n + n\frac{1}{n}\right)$$

$$= -\frac{1}{n^2 \ln n} - \frac{1}{(n \ln n)^2}$$

$f'(n) < 0 \Rightarrow f(n)$ is decreasing

and $f(n) > 0$ for $n \geq 2$.

$$\int_2^{\infty} \frac{1}{n \ln n} dn \qquad u = \ln n \quad du = \frac{1}{n} dn$$

$$= \int_{\ln 2}^{\infty} \frac{1}{u} du$$

$$= \ln|u| \Big|_{\ln 2}^{\infty}$$

$$= \lim_{n \to \infty} \ln n - \ln(\ln 2)$$

$$= \infty$$

$\int_2^{\infty} \frac{1}{n \ln n} dn$ diverges,

$\therefore \sum_{n=2}^{\infty} \frac{1}{n \ln n}$ diverges.

$$g(n) = \frac{1}{n(\ln n)^2} = n^{-1}(\ln n)^{-2}$$

$$g'(n) = -\frac{1}{n^2} \cdot \frac{1}{(\ln n)^2} + \frac{1}{n}\left(-\frac{2}{(\ln n)^3} \cdot \frac{1}{n}\right)$$

$$= -\frac{1}{(n \ln n)^2}\left(1 + \frac{2}{\ln n}\right)$$

$g(n) > 0$ and $g'(n) < 0$ for $n \geq 2$.

$$\int_2^{\infty} \frac{1}{n(\ln n)^2} dn \qquad u = \ln n \quad du = \frac{1}{n} dn$$

$$= \int^{\infty} \frac{1}{u^2} du$$

$$= -\frac{1}{u}\Big|_{\ln 2}^{\infty}$$

$$= -\left(0 - \frac{1}{\ln 2}\right)$$

$$= \frac{1}{\ln 2}$$

$\therefore \int_2^{\infty} \frac{1}{n(\ln n)^2} dn$ converges and

$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ converges.